Towards an Unbounded Implicit Arithmetic for the Polynomial Hierarchy

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Introduction

2 cousin approaches to logical characterization of complexity classes:

Bounded arithmetic (86 –)	Implicit computational complexity (91 –) (ICC)
	function algebras types/proofs-as-programs arithmetics / logics
large variety of complexity classes	resource-free characterizations of complexity classes (e.g. ramification)
corresp. with *proof-complexity*	extensions to programming languages

Introduction

"*f is provably total in system* XXX" $\Leftrightarrow$ $f \in$ complexity class YYY

Bounded arithmetic	Implicit computational complexity
Buss (S_1): $\forall x, \exists y \leq t.\ A_f(x, y)$	Leivant (intrinsic theories): $\forall x. N_1(x) \to N_0(f(x))$
FP, FPH	FP

A current limitation of ICC logics

- fewer complexity classes characterized by ICC logics than by bounded arithmetic
- in particular, not so satisfactory for non-deterministic classes e.g. NP, PH (polynomial hierarchy) ...

Recap : the polynomial hierarchy

- to be done: definition . . .

Our goal

design an unbounded arithmetic for characterizing FPH

expected benefits:

- bridge bounded arithmetic and ICC logics
- enlarge the toolbox ICC logics of , by exploring the power of quantification

Methodology

We want to use:

- ramification
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